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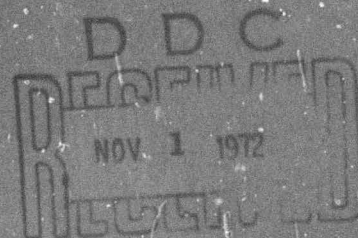
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BY

JOHN A. TOMLIN

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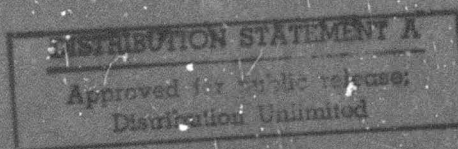
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In this note we show how the new updating techniques for triangular factors of the basis can be modified for the generalized upper bounding algorithm.

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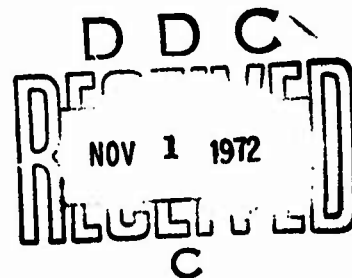
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Introduction

Two recent advances in linear programming have been the very successful implementation of the Generalized Upper Bound (GUB) algorithm, due to Dantzig and Van Slyke [3] and the new methods for updating triangular factors of the basis in the Simplex Method (Bartels [1], Forrest and Tomlin [4]). The purpose of this note is to show that despite the special basis inverse manipulation involved in one step of the GUB algorithm these two techniques can be successfully combined.

We use the notation and terminology of Beale [2], denoting the GUB problem as maximize x_0 subject to:

$$x_0 + \sum_{k=0}^{t} \sum_j a_{ojk} x_{jk} = b_0 \, ,$$

$$\sum_{k=0}^{t} \sum_j a_{ijk} x_{jk} = b_i \, , \quad (i = 1, \ldots, m) \qquad (1)$$

$$\sum_j x_{jk} = b_{m+k}, \quad (k = 1, \ldots, t)$$

We denote the key variable of each set by $x_{j_k k}$ and eliminate them to obtain the reduced system

$$x_0 + \sum_{k=0}^{t} \sum_j a^*_{ojk} x_{jk} = b^*_0$$

$$\sum_{k=0}^{t} \sum_j a^*_{ijk} x_{jk} = b^*_i \qquad (i = 1, \ldots, m) \qquad (2)$$

where

$$a^*_{ijk} = a_{ijk} - a_{ij_kk}$$

$$(i = 1, \ldots, m) \qquad (3)$$

$$b^*_i = b_i - \sum_{k=1}^{t} a_{ij_kk} b_{m+k}$$

and by convention $a_{ij_00} = 0$ since there is no key for the non-GUB variables.

The GUB algorithm now works with the reduced system (2) and its basis B. The modifications of the product form simplex algorithm required are detailed by Beale ([2], pp. 128-130).

Change of Basis

The only special feature of the GUB algorithm of concern here is the change of basis, and then only in one of the possible cases. This occurs when the incoming variable x_{qr} happens to eliminate the key variable $x_{j_s s}$ of some set s (where s may or may not equal r) which has some other non-key variables in the basis. In this case a new key must be found for set s, which we choose from among the basic non-key variables. Let the old and new keys be $x_{j_0 s}$, $x_{j_N s}$ and let $x_{j_u s}$ be the other non-key variables in the set. The standard product form method proceeds by observing that the old non-key columns $a^*_{\sim j_u s} = a_{\sim j_u s} - a_{\sim j_0 s}$ in the reduced basis B may be effectively replaced by their new representations $a^*_{\sim j_u s} = a_{\sim j_u s} - a_{\sim j_N s}$ through the identity

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$$\overset{*}{\underset{\sim}{a}}_{j_u}s = \underset{\sim}{a}_{j_u}s - \underset{\sim}{a}_{j_N}s = (\underset{\sim}{a}_{j_u}s - \underset{\sim}{a}_{j_0}s) - (\underset{\sim}{a}_{j_N}s - \underset{\sim}{a}_{j_0}s). \tag{4}$$

That is the new representation of the columns $\overset{*}{\underset{\sim}{a}}_{j_u}s$ in B may be obtained simply by multiplying B on the right by two-element column transformations.

This procedure works well for the standard product form. However if the basis is maintained in triangular factor form, i.e.,

$$B = LU \tag{5}$$

where L is lower and U is upper triangular, this technique leads to a loss of structure which makes further iterations all but impossible.

The Modified Technique

An alternative to using the elementary transformations referred to above is to carry out explicit column operations on B, though the amount of work required would make complete reinversion more attractive. Using the LU form of inverse however, we may operate on U much more conveniently. In the process of forming LU we will have pivoted on the columns $\overset{*}{\underset{\sim}{a}}_{j_u}s$ to produce

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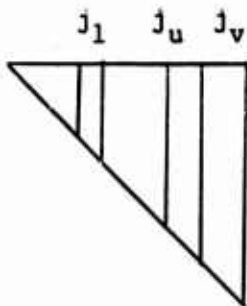
where the columns u_{j_u} $(u = 1, \ldots, v)$ of U correspond to the non-key basic columns of set s. Now if we choose j_N from among the j_u we see that multiplying the identity (4) on the left by L^{-1} we obtain

$$L^{-1} a^*_{j_u s} = u_{j_u} - u_{j_N} \qquad (j_u \neq j_N) \qquad (6)$$

Hence we choose $j_N = j_1$ to maintain triangularity and drop the new key from the basis. Similarly we must modify the "partially updated" incoming column [5] if it belongs to the same set (i.e., $r = s$) since our previous representation $y = L^{-1}(a_{qr} - a_{j_0 s})$ involves the old key. The new representation is from (4)

$$L^{-1} a^*_{qr} = y - L^{-1}(a_{j_N s} - a_{j_0 s})$$
$$= y - u_{j_N} \qquad (7)$$

This column is added to the right of U giving, for the new basis $\bar{B}$,

$$L^{-1}\bar{B} = H =$$
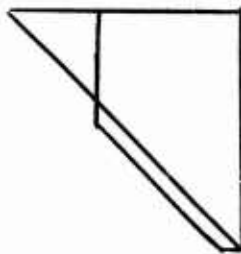
where H is upper Hessenberg, u_{j_N} is removed, and columns j_u $(\neq j_N)$ are replaced by $u_{j_u} - u_{j_N}$. This matrix H may now be reduced back to upper triangular form by any of the available methods (see [1],[4],[5]).

Note that if there is only one non-key basic variable in the set (i.e., $v = 1$) no subtraction of columns is necessary and the basis

updating procedure becomes identical to that of an ordinary simplex step. If there is more than one such column each $\underset{\sim}{u}_{j_u}$ must have $\underset{\sim}{u}_{j_N}$ subtracted from it.

Discussion

Although our choice of $j_N < j_u$ preserves triangularity we cannot choose j_N on the grounds of either sparsity or numerical stability and furthermore the modified columns may now have more non-zero entries. This makes repacking of the product form inverse of U necessary. This is not serious in the Bartels and Golub algorithm since new non-zero elements in U are generated anyway (see [1], [5]). However in the Forrest and Tomlin method [4] the whole point is to avoid creation of new non-zero elements in existing packed columns of U. Fortunately cases where repacking would be necessary seem to be very rare. Examination of a number of runs of GUB problems with the UMPIRE mathematical programming system show that basis changes of this type with $v > 1$ occur in only about two per cent of the iterations. This means that the time lost in repacking will be marginal and in fact we may take advantage of the opportunity to purge the backward transformation (U) file of deleted vectors and elements ([4], p. 272).

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